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Triangular Fuzzy Merec (TFMEREC) and Its Applications in Multi Criteria Decision Making

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Abstract

This paper presents a proposed method, Triangular Fuzzy MEREC (TFMEREC), which combines Triangular Fuzzy Numbers (TFNs) and Method Based on The Removal Effects of Criteria (MEREC). This integration aims to create an efficient implementation process that provides an efficient solution for complex MCDM issues, focusing on assessing halal suppliers. To demonstrate the feasibility and effectiveness of TFMEREC, this study will use an illustrative example to employ three different normalization methods with three different distance methods. Key findings from this research show that TFMEREC improves the accuracy and reliability of criteria weight determination. TFMEREC gives decision-makers more accurate weights, allowing for more informed decision-making processes. Furthermore, the sensitivity analysis provides insights into the impact of various normalization and distance methods on overall results, which improves the method's applicability and reliability. The TFMEREC method is a promising approach for dealing with imprecise and uncertain information in decision-making contexts, with potential applications in various domains. Overall, the findings underline the importance of methodological breakthroughs in enhancing decision-making processes, and the study is relevant to both experts and researchers.

Keywords: Multi-criteria decision-making, Objective weighing model, MEREC, Uncertainty and vagueness, Triangular fuzzy numbers, Halal supplier selection.

1 | Introduction

1.1 | Practical and Methodological Aims of the Study

In today's globalized market, ensuring the integrity of halal supply chains is critical for businesses catering to Muslim consumers. The process of assessing and selecting halal suppliers involves multiple criteria, including

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quality, compliance with religious guidelines, and overall reliability. However, this assessment is often challenged by imprecise and uncertain information, making effective decision-making difficult.

The complexity of Multi-Criteria Decision-Making (MCDM) in halal supplier assessment necessitates advanced methodologies that can handle uncertainty and provide accurate criteria weight determination. Traditional methods, while useful, often fall short of addressing the nuances of fuzzy and uncertain data. This limitation highlights a significant research gap in the existing literature on MCDM techniques applied to halal supplier evaluation.

Existing studies have employed various MCDM methods, such as the Analytic Hierarchy Process (AHP) [1] and Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) [2], to address supplier selection. However, these methods typically do not integrate fuzzy logic with criteria removal effects, which are crucial for handling ambiguity in decision-making. The next study by Hezam [3] proposes a decision-making model to assess service quality in Higher Education Institutions (HEIs). To achieve this, the study integrates various criteria related to service quality, including political, economic, social, technical, and environmental aspects. The MCDM methodology is employed, specifically using the Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEE) to rank alternative service quality measures.

Another study by Lazarashouri et al. [4], propose an innovative framework that combines simulation and fuzzy MCDM to enhance efficiency within Emergency Departments (EDs). The framework utilizes a Discrete Event Simulation (DES) model to capture the complex dynamics of ED operations. A study by Faghidian et al. [5] investigates the enablers of Total Quality Management (TQM) within the steel industry. It aims to understand the relationships between these enablers using the DEMATEL-ISM integration method, which combines Decision Making Trial and Evaluation Laboratory (DEMATEL) and Interpretive Structural Modeling (ISM). Farajpour [6] introduces an innovative approach to Multi-Attribute Decision-Making (MADM) with interval data. Traditional methods like TOPSIS and AHP are well-established for crisp data. However, this study adapts Support Vector Machine (SVM) for MADM with interval data. The SVM-based method is compared with interval TOPSIS, demonstrating its effectiveness.

Another study by Xu et al. [7], addresses the challenge of declining favorability toward an educational institution. To tackle this, the study proposes a novel methodology that combines hard and soft Operations Research (OR) approaches. The process involves problem structuring, MADM, and Data Envelopment Analysis (DEA). By integrating these tools, the study offers insights for institutions seeking adaptive strategies to enhance favorability and overcome enrollment declines. Another study by Jameel et al. [8] uses the TOPSIS method and spherical fuzzy set to evaluate climate change adaptation planning barriers. The model considers economic, social, technological, market, environmental, and time frame factors. The results show economic criterion is the most important, while social barriers are the least.

Edalatpanah [9], Akram [10], and Radha [11] discussed the impact of some applications of Pythagorean fuzzy in feature selection. Dirik and others [12–15] used Pythagorean fuzzy to develop decision models. Khalaf and Mohammed [16], Ali and Mohammed [17] combined the above model with developed topological concepts in topological spaces. Al-sharqi et al. [18] introduced the MCDM method for decision-making based on multi-mathematical structures like complex fuzzy structures [19], [20] fuzzy graph structures [21], [22] and some algebraic structures [23], [24]. Al-Quran et al. [25], [26] proposed approaches for the selection of MCDM technology by using the fuzzy set and its extension method,

Furthermore, a study by Jing et al. [27] focuses on the optimal selection of stock portfolios using MCDM methods for companies listed on the Tehran Stock Exchange. Various MCDM methods are employed, including the TOPSIS method, the taxonomy method, and other techniques like ARAS, VIKOR, COPRAS, and WASPAS. By integrating these methods, investors can make informed choices and identify the optimal stock portfolio with the highest return. Kordsofla et al. [28] study proposes an innovative approach by integrating the Matrix Approach to Robustness Analysis (MARA) with the well-established Strengths-Weaknesses-Opportunities-Threats (SWOT) matrix. The goal is to enhance strategic planning for travel

agencies; this approach provides a structured way for travel agencies to navigate uncertainties and improve decision-making processes. El-Araby's [29] study investigates the applicability of the Measurement Alternatives and Ranking according to the Compromise Solution (MARCOS) method in various engineering contexts. The MARCOS method is a novel MCDM approach designed to address issues such as the Rank Reversal Phenomenon (RRP) and inconsistent rankings observed in existing methods. The study compares MARCOS with other MCDM techniques and demonstrates its robustness.

On the other hand, a study by Li et al. [30] proposes an approach to evaluate and rank high schools using MCDM techniques. The study incorporates both quantitative and qualitative criteria, considering stakeholder perspectives. Key highlights include the selection of 10 criteria and 53 sub-criteria, the use of AHP and TOPSIS, and the application of fuzzy logic to handle expert judgments. In addition, a study by Akram et al. [31] explores properties and tasks related to Cq-RPFSs and proposes specialized aggregation operators for them. These include complex q-rung picture fuzzy Einstein averaging operators based on Einstein operations, weighted averaging, and ordered weighted averaging. The methodology is applied to solve MCDM problems, and a comparative study demonstrates its superiority and consistency.

Particularly, a study by Dehghani Filabadi et al. [32] proposes a novel approach for multi-period Multi-Attribute Group Decision-Making (MAGDM) using type-2 fuzzy sets of linguistic variables. The method addresses complex decision scenarios by considering both attribute evaluation information and attribute weights expressed in type-2 fuzzy terms. By leveraging type-2 fuzzy sets, the approach captures linguistic uncertainty and temporal aspects. Saberhoseini et al. [33] address the challenge of selecting the optimal private-sector Partner For Public-Private Partnerships (PPPs). These partnerships involve infrastructure and public service projects. The complexity arises from many indicators, imprecise judgments, and environmental unpredictability. The proposed algorithm simplifies this process by considering Critical Risk Factors (CRFs) and Key Performance Indicators (KPIs). Decision-makers' views are collected using Single-Valued Neutrosophic Sets (SVNSs), which handle ambiguous or incomplete information.

Abdel Aal et al. [34] propose an MCDM methodology for choosing the best charcoal company from various options and criteria. The study considers factors such as financial aspects, safety, labor, power supply, production, and transportation. The Combinative Distance Assessment (CODAS) method is used to rank alternatives and identify the optimal choice. By dividing criteria into positive and negative aspects, the study ensures robust decision-making. Sensitivity analysis confirms the stability of alternative rankings across different scenarios. Lastly a study by Colombo et al. [35], explore the intricate relationships between cryptocurrencies and external factors. Specifically, they investigate how financial indicators and social media indicators impact cryptocurrency behavior. The study introduces an innovative approach that combines entropy weighting and MADM to decipher these complex interplays. Therefore, there is a need for a more robust method that combines fuzzy logic with MCDM to improve decision accuracy and reliability.

To bridge this gap, we propose the TFMEREC method, which integrates Triangular Fuzzy Numbers (TFNs) with the method based on the Removal Effects of Criteria (MEREC). The research question guiding this study is: How can the integration of TFNs with MEREC improve the accuracy and reliability of criteria weight determination in the context of halal supplier assessment?

In this study, we aim to develop and validate the TFMEREC method through an illustrative example, employing three different normalization methods and three different distance methods. Our contributions include demonstrating the feasibility and effectiveness of TFMEREC in enhancing decision-making processes, particularly in the context of imprecise and uncertain information. We also conduct a sensitivity analysis to evaluate the impact of various normalization and distance methods on the overall results.

1.2 | Motivation for Conducting Research

The motivation for this research stems from the growing complexity and importance of halal supply chain management in the global market. As the demand for halal products increases, businesses face the challenge

of ensuring their suppliers adhere to stringent halal standards. This requirement necessitates a reliable and accurate decision-making process for selecting halal suppliers.

Current MCDM methods employed in supplier assessment often struggle with the inherent uncertainty and imprecision of data, leading to suboptimal decisions. Traditional approaches such as AHP and TOPSIS provide valuable frameworks but lack the ability to adequately handle fuzzy and ambiguous information, which is prevalent in real-world scenarios. This shortfall underscores the need for innovative solutions that can integrate fuzzy logic to enhance decision-making accuracy.

Additionally, the existing literature reveals a gap in methodologies that combine fuzzy logic with the effects of criteria removal, a critical aspect of robust decision-making. The lack of such integrated approaches highlights an opportunity to advance the field of MCDM by developing a method that addresses these limitations.

The introduction of TFMEREC aims to fill this gap by providing a method that improves the accuracy and reliability of criteria weight determination under uncertainty. By integrating TFNs with the MEREC method, we seek to offer a more comprehensive tool for decision-makers in the halal supply chain context.

This research is motivated by the potential to significantly enhance the decision-making process, leading to better outcomes in halal supplier selection and beyond. By addressing the limitations of current methods and contributing a novel approach to the field, this study aims to provide valuable insights and practical solutions for both researchers and practitioners involved in MCDM.

1.3 | The Contributions of the Study

This study makes several significant contributions to the field of MCDM, particularly in the context of halal supplier assessment. One of the primary contributions is the introduction of the TFMEREC method, which integrates TFNs with the MEREC. This novel integration enhances the handling of imprecise and uncertain information, providing a more accurate and reliable framework for decision-making.

The TFMEREC method improves the accuracy and reliability of criteria weight determination by incorporating fuzzy logic into the MEREC method. This enhancement allows decision-makers to derive more precise weights, which are crucial for making informed decisions in complex MCDM problems. Furthermore, by applying TFMEREC to the specific context of halal supplier assessment, the study demonstrates the practical relevance and effectiveness of the method in addressing real-world problems, providing a valuable tool for businesses operating in the halal market.

In addition to presenting the TFMEREC method, the research includes a comprehensive analysis using three different normalization methods and three different distance methods. This comparison provides insights into how these methods impact the overall results, offering guidance on selecting appropriate techniques for different decision-making scenarios. The study also conducts a sensitivity analysis to evaluate the robustness of the TFMEREC method, helping to understand the impact of varying parameters on the results and enhancing the method's reliability and applicability in diverse contexts.

By addressing the limitations of existing MCDM methods and introducing a new approach that integrates fuzzy logic with criteria removal effects, this study contributes to the advancement of MCDM methodologies. The findings highlight the importance of methodological innovations in improving decision-making processes. Overall, the study provides a comprehensive solution to the challenges of decision-making under uncertainty, offering a robust and reliable method for evaluating halal suppliers. The contributions extend beyond the specific application, presenting a framework that can be adapted and applied to various domains where MCDM is critical.

1.4 | Organization of the Paper

Following this study, we shall focus on the MEREC approach. The MEREC technique assesses how removing each criterion affects the overall performance of alternatives. Criteria with less influence receive

lower weights. Eliminating a criterion with a higher weight causes a greater change in the ranking of alternatives compared to one with a lower weight. This reflects the principle that the importance of a criterion is proportional to its impact on the decision. The MEREC method allows for excluding unimportant factors from the evaluation process as needed, making the decision-making process more efficient and focused. Few papers utilized the MEREC approach, indicating that it is a relatively new and underexplored method in the field of MCDM.

2 | Preliminaries

2.1 | Triangular Fuzzy Number

TFNs are widely acknowledged in the profession as a crucial tool for dealing with the ambiguities and inconsistencies that arise in Multiple Criteria Decision Making (MCDM). The literature extensively acknowledges the usefulness of TFNs in mimicking uncertainty and subjective evaluations due to their flexible and intuitive nature. This review provides a comprehensive analysis of the main contributions, methodological improvements, and practical uses of TFNs in the field of MCDM.

Zadeh [36] established the notion of fuzzy numbers in 1965, which provided the fundamental basis for applying fuzzy sets in decision making. TFNs have the potential to accurately portray the inherent uncertainty and vagueness that are present in many decision-making processes. TFNs improve decision-making frameworks in Multi-Criteria Decision Making (MCDM) approaches, which are frequently employed in the AHP and TOPSIS procedures, as referenced in academic literature.

Faizi et al. [37] suggest a novel approach for decision-making in uncertain contexts by combining the COMET methodology with normalization interval-valued TFNs. This approach manages uncertainty and lends support to decision-making procedures by combining the use of TFNs with other fuzzy number representations. Al Mohamed et al. [38] proposed a hybrid Fuzzy Multi-Criteria Decision Model (FMCDM) to find the optimal location based on a combination of factors. The Fuzzy Analytical Hierarchy Process (FAHP) was used to estimate the relative criteria classification through the evaluation process. Then, the Fuzzy Technique of Order Preference Using Similarities to the Perfect Solution (FTOPSIS) was applied to rank the possible alternative sites.

Diaz et al. [39] defined an admissible order on TFNs and studied some fundamental properties of their arithmetic and their relationship with this admissible order. It also introduced the concepts of average function on TFNs and studied a generalized structure of the vector spaces. Abdullah and Lim [40] aimed to unravel the relationships between criteria of e-commerce using the fuzzy DEMATEL, where TFNs were used to represent uncertain and ambiguous information. The main contribution of this paper was the development of the causal relationship among the criteria of e-commerce that were obtained from a total relation matrix of the fuzzy DEMATEL.

Although valuable insights have been generated through the application of TFNs in MCDM, challenges such as the computational complexity and subjectivity associated with defining TFN parameters continue to persist. Additional research should be dedicated to the development of computational methods that are more resilient and efficient in the management of TFNs. Additionally, it is recommended that an investigation be undertaken regarding the incorporation of machine learning methodologies to simplify and enhance the calculation of TFN parameters. In conclusion, it will be critical to maintain ongoing research and development efforts in this field to surmount the obstacles linked to TFNs and guarantee their effective implementation in MCDM.

2.1.1 | Normalization techniques of TFNs

Normalization techniques are used in MCDM to convert the raw data into a comparable scale. This ensures that all the criteria are given equal importance when deciding. Some normalization techniques used in MCDM are linear normalization, vector normalization, min-max normalization, and standardization [41].

In recent years, a substantial number of studies have been conducted on the effect of normalization approaches on MCDM, and a study evaluated the applicability of several formulations of normalizing procedures inside MCDM to the final design selection based on the discovered Pareto frontier. According to the study's findings, different normalization formulas are used to shift the solution values of the Pareto frontier to the needed data range for input to MCDM algorithms [42]. A comparative study examines the influence of normalization approaches on decision-making regarding internal ship design problems. The outcomes are evaluated by comparing the top-ranked alternatives and calculating the correlation coefficients of the obtained rank for each alternative. Methods of normalization that are able to preserve the dominant order of alternatives resulted in comparable final design selections.

Another study investigated several normalizing strategies for MCDM using a case study involving selecting an appropriate location for a solar power facility [43]. The study concluded that different normalization procedures can produce varying results, and it is crucial to choose the most effective methodology for the given scenario.

In MCDM, linear normalization is one of the most prevalent and straightforward techniques. It converts the primary data to a standard scale that runs from 0 to 1, and it does this automatically [44]. This is done by subtracting the minimum value and dividing it by the range of the data. The simplicity of this method and its ease of computation are among its primary benefits. However, it may not be suitable when the data contains extreme values or outliers, as these can distort the normalized values. In the context of MEREC, linear normalization is a viable option because of how straightforward it is and how straightforward it is to compute. It offers a convenient method for transforming a variety of criteria onto a standard scale, which is an essential stage in the process of aggregating criteria in MCDM.

Vector normalization is another approach used in MCDM. It requires dividing each value by the magnitude of the vector to which it belongs. The magnitude of a vector is calculated as the square root of the sum of the squares of its components. This approach assures that the normalized data keeps the same direction as the original data and its magnitude (length) is 1 [45]. In MEREC, vector normalization could be chosen since it maintains the direction of the original data. This could be relevant in instances where the relative orientation of the criterion in the multi-dimensional space is important.

A method known as sum linear normalization involves dividing each value by the total of all the values. This guarantees that the normalized data totals up to 1, which is the desired result [46]. When the data being analyzed comprises sections of a whole, this approach shines as a very effective option. When the criteria represent pieces of a whole, sum linear normalization is a method that might be used in the context of MEREC. For example, if the criteria are percentages representing the allocation of resources, then sum linear normalization would be a suitable choice.

In conclusion, the choice of normalization technique in MCDM, specifically in MEREC, depends on the data's nature and the decision-making problem's unique requirements. This is true both generally and specifically in MEREC. In MEREC, this study chooses linear normalization, vector normalization, or sum linear normalization because of the various advantages they provide in handling proportional data, preserving data direction, and simplifying the process. The following are MCDM normalizing applications.

2.1.2 | Distance techniques of TFNs

This section will go over the definitions of the Euclidean distance, Separation distance, and Hamming distance, which apply to MCDM.

A difficult procedure, MCDM requires the evaluation of several contradictory criteria in order to reach an informed conclusion. MCDM relies heavily on distance approaches to ascertain the optimal alternative through the evaluation of multiple criteria during the selection process [47]. The selected distance method can have a substantial influence on the outcomes of the MCDM procedure. This study shall explore the rationale behind the selection of Euclidean distance, separation distance, and Hamming distance in the MEREC MCDM approach.

Distance techniques based on the Euclidean distance are frequently employed in MCDM techniques such as TOPSIS [48]. Its definition is the length of the line segment connecting two points. It evaluates the geometric distance between an alternate solution and an ideal or anti-ideal solution in the criteria space within the context of MCDM. By employing Euclidean distance in MCDM, the degree of similarity or dissimilarity between alternatives is measured in a basic and intuitive manner. It is especially beneficial when the criteria are continuous, and the difference between criteria values is significant.

In MCDM, separation distance, which is often referred to as the Minkowski distance, is an additional crucial distance metric [42]. It permits the contribution of each dimension to the overall distance to be controlled by a variable exponent, constituting a generalization of the Manhattan distance and Euclidean distance. The separation distance in MCDM quantifies the extent of deviation between a given alternative solution and an ideal or anti-ideal solution. The use of separation distance in MCDM enables a versatile and adjustable metric for assessing the degree of similarity or dissimilarity among alternatives, hence catering to various criterion types and contexts of decision-making.

The Hamming distance, which measures the number of points where the corresponding values differ, is a distance metric. It can be utilized to quantify the dissimilarity between binary or categorical criteria within the framework of MCDM. Hamming distance is a straightforward and efficient metric utilized in MCDM to assess the degree of similarity or dissimilarity between options in cases where the criteria are categorical or binary. This is especially beneficial in cases where the criteria pertain to qualitative qualities [47-49].

This paper presents the TFMEREC model as an effective approach to dealing with uncertainties and vagueness in decision-making for selecting Halal suppliers. The selection of Halal suppliers is crucial for maintaining the halal integrity of the food supply chain [50]. The halal status of a product is not only determined by the ingredients used but also by the processes involved in its production, handling, and distribution. Therefore, it is essential to ensure that all suppliers in the supply chain comply with Halal standards. The TFMEREC model provides decision analysts with a more systematic and effective decision support tool, enabling them to understand the complete evaluation process better and provide a more feasible and effective solution.

By incorporating uncertainty and vagueness into the decision-making process, the TFMEREC model allows decision-makers to make more informed and confident decisions, thereby enhancing the quality and credibility of the decision. Furthermore, by providing a clear and transparent evaluation process, the TFMEREC model facilitates communication and consensus-building among stakeholders, contributing to the successful implementation of the decision. Thus, the TFMEREC model represents a significant advancement in the field of MCDM and has the potential to make a substantial impact on decision-making practice in various domains. This study hopes to stimulate additional research and application of the TFMEREC model, as well as contribute to the ongoing development and refinement of MCDM techniques.

The importance of considering both halal and food safety standards when choosing food suppliers is also emphasized. In the food industry, maintaining food safety is essential, and compromising this can have detrimental effects on the general public's health [51]. So, it's crucial to choose vendors who can adhere to Halal and food safety regulations. The study by Mabkhot and Hashed [51] looks into the impact of several factors on the long-term performance of halal products in Malaysia's food industry. While it does not use the term delivery timeliness, it does highlight Supply Chain Integration (SCI), which is indirectly related to timely delivery.

Unfortunately, the term nonconformance rate is not explicitly addressed in the accessible references. However, in the larger context of halal product performance, factors including Process Quality Improvement (PQI) and Food Safety Concerns (FSCs) may indirectly influence nonconformance rates. The same article emphasizes the significance of PQI in sustaining product performance. PQI improves product quality in the halal food business. While the given references do not specifically include pricing, it is crucial to consider both halal and food safety criteria when choosing food providers. The Kuala Lumpur Convention Centre promotes continual development in-service performance, including food delivery, in terms of quality and

halal requirements [52]. In summary, the importance of adhering to both halal and food safety regulations when selecting food suppliers is emphasized. Ensuring compliance not only supports the halal industry but also safeguards public health.

Finally, we will validate the TFMEREC using two subsections: comparative analysis and sensitivity analysis, which will demonstrate the feasibility and effectiveness of the proposed method. Comparative analysis involves comparing the TFMEREC model results with those obtained using other MCDM methods. This allows us to evaluate the performance of the TFMEREC model and identify its advantages and limitations. Sensitivity analysis, on the other hand, involves examining how changes in the input parameters (such as the weights of the criteria) affect the outcome of the decision. The result reveals the decision's robustness and the TFMEREC model's reliability.

2.2 | Fundamentals of TFNs

TFNs and their characterizing conditions have been defined. The operational principles and degree of possibility of TFNs are explained.

Definition 1. Let $\tilde{s}=[s_l,s_m,s_r]$. A fuzzy number $\tilde{X}$ or M is said to be a TFN if its membership function $\mu_{\tilde{x}}:M\rightarrow[0,1]$ has the following conditions [53]:

I. $\tilde{X}$ is convex for all $x_1,x_2\in M$ and $\lambda\in[0,1]$ $\mu_{\tilde{X}}(\lambda_{x1}+(1-\lambda)x_2)\geq\min\{\mu_{\tilde{x}}(x_1),\mu_{\tilde{x}}(x_2)\}$.

II. $\tilde{X}$ is normal, which means that there exists an $x\in M$ such that $\mu_{\tilde{x}}(x)=1$.

III. $\tilde{X}$ is piecewise continuous.

Next, we will show the TFN membership function and proof.

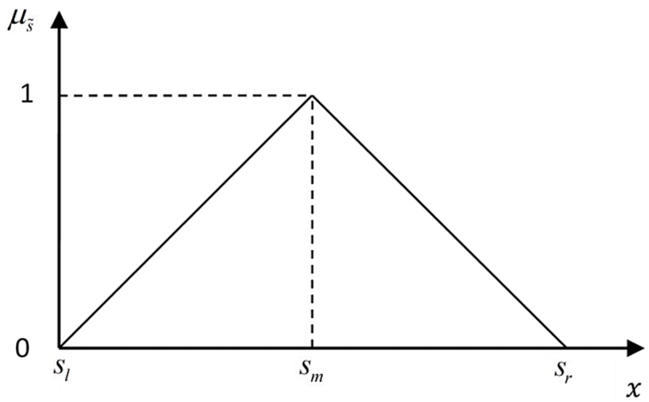

Fig. 1. Membership function.

Now, if we get a crisp interval by α-cut operation, interval $\tilde{S}_a$ shall be obtained as for all $_{\tilde{s}}\in[0,1]$.

Proof: from

$$\frac{s_l^{(\alpha)}-s_l}{s_m-s_l}=\alpha,\frac{s_r-s_r^{(\alpha)}}{s_r-s_m}=\alpha.$$

We get,

$$s_l^{(\alpha)}=(s_m-s_l)\alpha+\alpha_l,$$

$$s_r^{(\alpha)}=-(s_r-s_m)\alpha+\alpha_r.$$

Thus

$$M_\alpha = \left[\alpha_l^{(\alpha)}, \alpha_r^{(\alpha)}\right] = \left[(s_m - s_l)\alpha + \alpha_l, -(s_r - s_m)\alpha + \alpha_r\right].$$

To conclude, *Eq. (1)* is a membership function

$$\mu_{\tilde{x}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, \text{for } a_1 \le x \le a_2 \\ = 1, \text{for } x = a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, \text{for } a_2 \le x \le a_3 \\ = 0, \text{ otherwise} \end{cases}. \tag{1}$$

Definition 2. A fuzzy number is a fuzzy set specified by $A = \{x, \mu A(x)\}$ with membership function μ_A and X taking its number on the real line, and it has the following attributes:

I. A continuous mapping to the closed interval [0, 1].

II. Constant on. $(-\infty, s]$: $\mu_A(x) = 0$ for all $x \in (-\infty, s]$.

III. Strictly increasing on $[s, t]$.

IV. Constant on. $[t, u]$: $\mu_A(x) = 1$ for all $x \in [t, u]$.

V. Strictly decreasing on $[u, d]$.

VI. Constant on $[d, \infty)$: $\mu_A(x) = 0$ for all $x \in [d, \infty)$.

2.3 | The Distance Techniques of TFNs

The three distance techniques, which are described below:

Definition 3. Let's assume two points, such as (a_1, b_1, c_1) and (a_2, b_2, c_2) in the two-dimensional coordinate plane for Euclidean distance [54].

$$V_E = \sqrt{\frac{1}{x}((a_2 - a_1)^2 + (b_2 - b_1)^2 + (c_2 - c_1)^2)}. \tag{2}$$

Definition 4. Let's assume two points, such as (a_1, b_1, c_1) and (a_2, b_2, c_2) in the two-dimensional coordinate plane for Separation distance [54].

$$V_S = \sqrt{(a_2 - a_1)^2 + (b_2 - b_1)^2 + (c_2 - c_1)^2}. \tag{3}$$

Definition 5. Let's assume two points, such as (a_1, b_1, c_1) and (a_2, b_2, c_2) in the two-dimensional coordinate plane for Hamming distance [48].

$$V_H = |(a_2 - a_1) + (b_2 - b_1) + (c_2 - c_1)|. \tag{4}$$

2.4 | The Normalization Techniques

The three normalization techniques which is described below:

Definition 6. Let where x_{ij} is the value of criterion j in alternative i, $\min(x)$ and are the $\max(x)$ minimum and maximum values of x across all options, and n_{ij} are the normalized benefit (B) and cost values (C), respectively. Linear Normalization is conducted as below [55]:

$$n_{ij} = \frac{x_{ij}}{\max\left(x_{ij}\right)} \in B. \tag{5}$$

$$n_{ij} = 1 - \frac{x_{ij}}{\max\left(x_{ij}\right)} \in C. \tag{6}$$

Definition 7. Let where x_{ij} is the value of criterion j in alternative i, $\min(x)$ and are the $\max(x)$ minimum and maximum values of x across all options, and n_{ij} are the normalized benefit (B) and cost values (C), respectively. Vector Normalization is operated as below [56]:

$$n_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^{2}}} \in B. \tag{7}$$

$$n_{ij} = 1 - \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^{2}}} \in C. \tag{8}$$

Definition 8. Let where x_{ij} is the value of criterion j in alternative i, $\min(x)$ and are the $\max(x)$ minimum and maximum values of x across all options, and n_{ij} are the normalized benefit (B) and cost values(C), respectively. Sum-Linear Normalization is shown below [56]:

$$n_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}} \in B. \tag{9}$$

$$n_{ij} = \frac{\frac{1}{x_{ij}}}{\sum_{i=1}^{m} \frac{1}{x_{ij}}} \in C. \tag{10}$$

2.5 | Method Based on the Removal Effects of Criteria

MEREC is conducted as below [57]:

Step 1. The decision matrix is constructed. Each row and column in the matrix will have a performance value associated with the corresponding alternative and criterion. The matrix will be a n x m matrix, where n is the number of alternatives and m is the number of criteria. If there are n alternatives and m criteria, the elements of this matrix are represented by x_{ij}, and the decision matrix has the following form:

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{ij} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2j} & \cdots & x_{2m} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{i1} & x_{i2} & \cdots & x_{ij} & \cdots & x_{im} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nj} & \cdots & x_{nm} \end{bmatrix}. \tag{11}$$

Step 2. The decision matrix has been normalised. It has been normalised in the choice matrix. The symbols n_{ij}^{x} are used to denote the components of the normalizing matrix. If B represents the collection of benefit criteria and C represents the collection of cost criteria, *Eq. (3)* may be used to normalize, the following formula is used to normalize the choice matrix:

$$n_{ij}^x = \begin{cases} \dfrac{\min\limits_k x_{kj}}{x_{ij}} & \text{if } j \in B, \\ \dfrac{x_{ij}}{\max\limits_k x_{kj}} & \text{if } j \in C. \end{cases} \quad (12)$$

Step 3. Make a logarithmic calculation of the alternatives' performance (S_i). Determine the effectiveness of the alternatives using a logarithmic calculation (S_i). In this stage, a logarithmic scale with equal weights for the criteria is employed to assess the overall performances of the alternatives. Using the normalized data from the previous stage, we can ensure that smaller values of n_{ij}^x lead to higher values of performance (S_i).

$$S_i = \ln\left(1 + \left(\frac{1}{m}\sum_j \left|\ln(n_{ij}^x\right|\right)\right). \quad (13)$$

Step 4. Remove each criterion from the alternatives' performance to calculate it. This study employs the logarithmic measure in this stage, just like the previous one. In contrast to *Step 3*, in this stage, each criterion is removed individually to determine how well the alternatives perform. There are, therefore, m sets of performances that match m criteria. If the performance of the ith alternative with respect to removing the ith criterion is represented by $S_{ij}^{'}$.

$$S_{ij}^{'} = \ln\left(1 + \left(\frac{1}{m}\sum_{k,k\neq j} \left|\ln(n_{ij}^x\right|\right)\right). \quad (14)$$

Step 5. Calculate the total of the absolute deviations. In this step, it calculates the elimination effect of the ith criterion using the values from *Steps 3* and *4*. Let E_j represents the outcome of removing the ith condition. E_j is calculated by computing the absolute deviation and adding it up.

$$E_j = \sum_i \left|S_{ij}^{'} - S_i\right|. \quad (15)$$

Step 6. Calculate the objective weights of criteria using the removal effects (E_j) values that were derived in the previous step. In the statements that follow, w_j signifies the weight of the ith criterion. The following formula is used to calculate how much each criterion is weighted:

$$W_j = \frac{E_j}{\sum_k E_k}. \quad (16)$$

3 | Research Framework

We present a research framework on the application of MCDM, MEREC, and TFN in addressing complex decision-making problems. Despite extensive research in these areas, the ambiguity inherent in MCDM has not been effectively addressed. To tackle this issue, we propose a novel method that combines TFN and MEREC, referred to as TFMEREC. The validation of this approach involves the application of three normalization methods and three distance techniques. This thorough process ensures a robust and reliable solution to the MCDM problem. The following figure will delve into the details of this innovative approach.

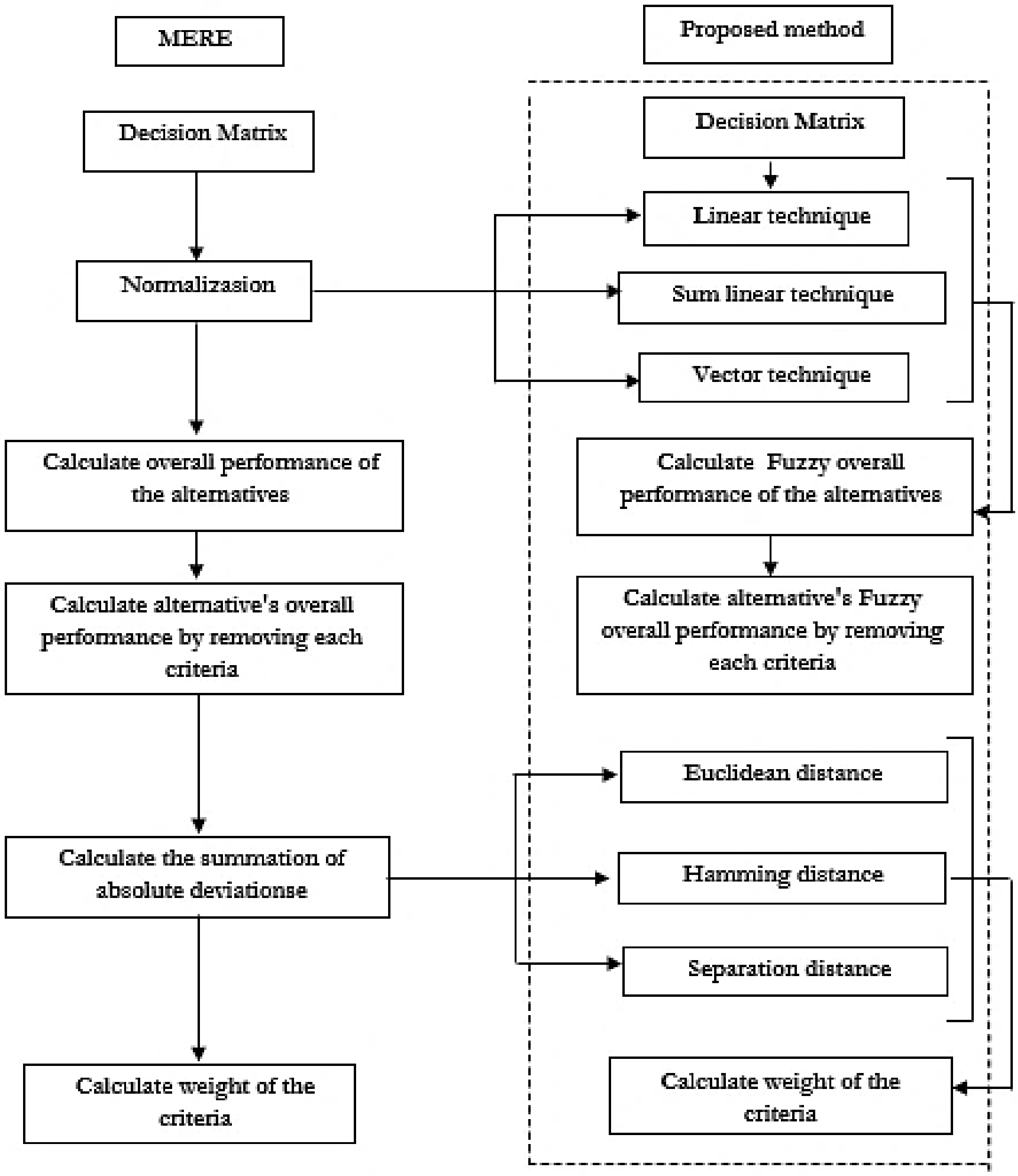

Fig. 2. Overall framework.

Fig. 2 outlines a framework that begins with acquiring knowledge from prior studies to gain a comprehensive understanding of MCDM, MEREC, and fuzzy numbers. Despite the wealth of existing research, they have yet to address the ambiguity inherent in the MCDM problem effectively. A novel method combining TFN and MEREC is proposed to tackle this uncertainty. The validation of this approach involves the application of three normalization methods (linear technique, sum linear technique, and vector technique) and three distance techniques (euclidean distance, separation distance, and hamming distance) within the context of TFMEREC. This process ensures a robust and reliable solution to the MCDM problem.

4 | The Proposed Triangular Fuzzy MEREC (TFMEREC)

The TFMEREC method combines the concepts of triangular fuzzy set theory and the MEREC method to aid decision-making in fuzzy environments. Its goal is to provide a reliable and comprehensive approach to assessing criteria and determining weights in MCDM scenarios.

To achieve this, the decision matrix is transformed using linguistic variables represented as TFNs. Normalization techniques are employed to ensure consistency and comparability. The effectiveness of

alternatives is evaluated using fuzzy arithmetic, and the performance of alternatives is calculated by removing each criterion. Specific distance functions are used to compute the distances between alternatives.

Ultimately, the weights of criteria are determined, allowing for a systematic evaluation of the decision problem. Here are the proposed steps of TFMEREC:

Step 1. The assessment of criteria is conducted using linguistic variables, which are then represented using TFNs.

Step 2. Create the decision-making matrix.

The elements of this matrix are denoted by the symbol x_{ij}, and they must be greater than zero ($x_{ij} > 0$). Since these values will be used in subsequent calculations and only the real natural logarithm function $\ln(x)$ is specified for $x_{ij} > 0$. This means that the natural logarithm of zero is unknown [58].

Step 3. Normalize the fuzzy decision matrix.

Normalization aims to decrease the difference in size among attributes and dimensions, ensuring that the normalized value is within the range of 0 to 1. Consequently, this process helps eliminate technical problems caused by different measurement components [59], [60]. In order to normalize the decision matrix, *Eqs. (17)*-(22), are employed. This normalization process involves converting the cost criteria to benefit criteria and vice versa. Applying these equations transforms the decision matrix to ensure consistency and comparability across different criteria. The proposed method will use three normalization techniques, which are described below:

Definition 9. Let where $x_{ij} = \left(a_{ij}^{l}, a_{ij}^{m}, a_{ij}^{u}\right)$ is the value of criterion j in alternative i, $\min(x)$and are the $\max(x)$ minimum and maximum values of x across all options, and n_{ij} are the normalized benefit and cost values, respectively. If B shows the set of beneficial criteria, and C represents the cost criteria, by utilizing the following equation for linear technique normalization when applying it to a triangular fuzzy number [58]:

$$n_{Lij} = \frac{\min\ a_{ij}^{l}}{a_{ij}^{u}}, \frac{\min\ a_{ij}^{l}}{a_{ij}^{m}}, \frac{\min\ a_{ij}^{l}}{a_{ij}^{l}},\ j \in B. \tag{17}$$

$$n_{Lij} = \frac{a_{ij}{}^{l}}{\max\ c_{ij}{}^{l}}, \frac{a_{ij}{}^{m}}{\max\ c_{ij}{}^{m}}, \frac{a_{ij}{}^{l}}{\max\ c_{ij}{}^{l}}, j \in C. \tag{18}$$

Definition 10. Let where $x_{ij} = \left(a_{ij}^{l}, a_{ij}^{m}, a_{ij}^{u}\right)$ is the value of criterion j in alternative i, $\min(x)$and are the $\max(x)$ minimum and maximum values of x across all options, and n_{ij} are the normalized benefit and cost values, respectively. If B shows the set of beneficial criteria, and C represents the cost criteria, by utilizing the following equation for vector technique normalization when applying it to a triangular fuzzy number [47]:

$$n_{Vij} = \frac{a_{ij}^{l}}{\sqrt{\sum_{i=1}^{m} x_{ij}^{2}}}, \frac{a_{ij}^{m}}{\sqrt{\sum_{i=1}^{m} x_{ij}^{2}}}, \frac{a_{ij}^{u}}{\sqrt{\sum_{i=1}^{m} x_{ij}^{2}}}\ ; j \in B. \tag{19}$$

$$n_{Vij} = 1 - \frac{a_{ij}^{u}}{\sqrt{\sum_{i=1}^{m} x_{ij}^{2}}}, 1 - \frac{a_{ij}^{m}}{\sqrt{\sum_{i=1}^{m} x_{ij}^{2}}}, 1 - \frac{a_{ij}^{l}}{\sqrt{\sum_{i=1}^{m} x_{ij}^{2}}};\ j \in C. \tag{20}$$

Definition 11. Let where $x_{ij} = \left(a_{ij}^{l}, a_{ij}^{m}, a_{ij}^{u}\right)$ is the value of criterion j in alternative i, $\min(x)$ and $\max(x)$ are the minimum and maximum values of x across all options, and n_{ij} are the normalized benefit and cost values, respectively. If B shows the set of beneficial criteria, and C represents the cost criteria, by utilizing the following equation for sum linear technique normalization when applying it to a triangular fuzzy number [47]:

$$n_{SLij}=\frac{a_{ij}^{l}}{\sum_{i=1}^{m}x_{ij}},\frac{a_{ij}^{m}}{\sum_{i=1}^{m}x_{ij}},\frac{a_{ij}^{u}}{\sum_{i=1}^{m}x_{ij}};\ j\in B. \tag{21}$$

$$n_{SLij}=\frac{\frac{1}{a_{ij}^{u}}}{\sum_{i=1}^{m}\frac{1}{x_{ij}}},\frac{\frac{1}{a_{ij}^{m}}}{\sum_{i=1}^{m}\frac{1}{x_{ij}}},\frac{\frac{1}{a_{ij}^{l}}}{\sum_{i=1}^{m}\frac{1}{x_{ij}}};\ j\in C. \tag{22}$$

Step 4. Determine the alternatives' overall performance.

In this step, the improved logarithm function is used to determine the overall performance of the alternatives. It is derived from a non-linear function pioneered by Shannon [61] and versions have been researched [62–64]. Following this, the proposed research will develop a new formula for triangular fuzzy alternatives overall performance and the computation is carried out using the following *Eq. (23)*.

Definition 12. Let m sets of performances and n criteria, $x_{ij}=\left(a_{ij},b_{ij},c_{ij}\right)$, and Q_{ij} indicate the alternatives' overall performance of ith alternative to the elimination of the ith criterion. The computation of this step is made using the following equation:

$$Q_i=\left(\ln\left(1+\left(\frac{1}{m}\sum_{k,k,\neq j}|\ln(c_{ik}^{x})|\right)\right),\ln\left(1+\left(\frac{1}{m}\sum_{k,k,\neq j}|\ln(b_{ik}^{x})|\right)\right),\ln\left(1+\left(\frac{1}{m}\sum_{k,k\neq j}|\ln(a_{ik}^{x})|\right)\right)\right). \tag{23}$$

Eq. (23), also known as Triangular Fuzzy Alternatives Overall Performance, is a new definition obtained by integrating TFN with *Eq. (13)*.

Now, this equation aims to verify that this definition is in TFN; therefore let

$$s_l=\ln\left(1+\left(\frac{1}{m}\sum_{k,k,\neq j}|\ln(c_{ik}^{x})|\right)\right),s_m=\ln\left(1+\left(\frac{1}{m}\sum_{k,k,\neq j}|\ln(b_{ik}^{x})|\right)\right),s_r=\ln\left(1+\left(\frac{1}{m}\sum_{k,k\neq j}|\ln(a_{ik}^{x})|\right)\right).$$

Hence,

$$Q_i=\left(s_l,s_m,s_r\right).$$

To prove the definition, first to prove $Q=\left(u_{ij},v_{ij},w_{ij}\right)$ is between 0 to 1 if U less than V, $\ln\left(|u|\right)$ is greater than $\ln\left(|v|\right)$.

Proof: since $u<v$ and $\ln(|u|)>\ln(|v|)$, it implies that $|u|>|v|$, and consequently, a and b are both negative. As a and b are negative, for (u,v,w) to be between 0 and 1, c must be positive.

Considering c as positive, it must fall between a and b, thereby satisfying the condition that (u,v,w) is between 0 and 1. Therefore, with $u<v$ and $\ln(|u|)>\ln(|v|)$, it's evident that (u,v,w) lies within the range of 0 to 1.

Next, to prove that u, v, and w are all between 0 and 1 and that their mean value is also between 0 and 1, given $u<v$ and $\ln(|u|)>\ln(|v|)$, it can follow these steps.

Proof: since $u<v$ and $\ln(|u|)>\ln(|v|)$, it concludes that u and v are negative and lie between 0 and 1. Additionally, any value w that lies between u and v must also be between 0 and 1. Considering the mean value $\frac{u+v+w}{3}$, since $u<0<v$ and $0<w<1$, the mean value will be a combination of these values, ensuring it

falls between 0 and 1. Thus, both the individual values u, v, w and their mean value fall within the range of 0 to 1.

So, to illustrate that the study has developed a new definition, *Eqs. (13)* and *(14)* have been combined with TFNs. We will show how to prove these equations using three TFN membership function criteria.

Proof: Q is convex. Let Q denote a fuzzy set on M and assume Q is convex.

This implies that because,

$\Rightarrow \alpha -$ cuts are convex.

To prove, If $x_1, x_2 \in \alpha_Q \Rightarrow \lambda x_1 + (1-\lambda)x_2 \in \alpha_Q$ and $\lambda \in [0,1]$, $Q(x_1) \le Q(x_2)$ and $\alpha = Q(x_1)$,

$$Q(\lambda x_1 + (1-\lambda)x_2) \ge \alpha = Q = \min[Q(x_1), Q(x_2)],$$

$$\Rightarrow Q(\lambda x_1 + (1-\lambda)x_2) \ge \min[Q(x_1), Q(x_2)] \text{ for all } x_1, x_2 \in M \text{ and for all } \lambda \in [0,1]$$

Proof: Q is normal.

Boundedness

Given a TFN, let Q be defined by the parameters (u_{ij}, v_{ij}, w_{ij}), where u represents the lower bound, w represents the upper bound, and v represents the middle bound value of the fuzzy number; its membership function $\mu_Q(x)$ is given by:

$$\mu(x) = \begin{cases} 0 & \text{if } x \le u, \\ \dfrac{x-u}{v-u} & \text{if } u \le x \le v, \\ \dfrac{w-x}{w-v} & \text{if } v \le x \le w, \\ 0 & \text{if } x \ge w. \end{cases}$$

Firstly, for $x \le u$ or $x \ge w$, $\mu_Q(x) = 0$, which satisfies $0 \le \mu_Q(x) \le 1$. Secondly, $u \le x \le v, \mu_Q(x) = \dfrac{x-u}{v-u}$. Since v is the maximum value in the interval $[u, v], \dfrac{x-u}{v-u}$ is always non-negative and less than or equal to 1. Finally, $v \le x \le w, \mu_Q(x) = \dfrac{w-x}{w-v}$. Since v is the maximum value in the interval $[v, w], \dfrac{w-x}{w-v}$ is always non-negative and less than or equal to 1. Thus $0 \le \mu_Q(x) \le 1$, for all x.

Middle bound value

The membership function $\mu_Q(x)$ is 1 at $x = v$ (within the interval $[u, v]$ or $[u, v]$ because it's the peak of the triangular fuzzy number.

Since both conditions are satisfied, therefore Q is normal.

Proof: Q is piecewise continuous.

Proof: given a TFN, let Q be defined by the parameters (u_{ij}, v_{ij}, w_{ij}), where u represents the lower bound, w represents the upper bound, and *v* represents the middle bound value of the fuzzy number; its membership function $\mu_Q(x)$ is given by

$$\mu(x)=\begin{cases}0 & \text{if } x\le u,\\ \dfrac{x-u}{v-u} & \text{if } u\le x\le v,\\ \dfrac{w-x}{w-v} & \text{if } v\le x\le w,\\ 0 & \text{if } x\ge w.\end{cases}$$

We can observe that within each interval $[(u,v),\ (u,v),\ \text{and}\ (w,\infty)]$ the membership function is defined by a linear function of *x*. Linear functions are continuous everywhere, so the membership function is continuous within these intervals.

At the boundary points u, v, and w, the membership function undergoes a change. However, these changes are controlled and bounded. Specifically, at $x=u$, there's a step change from 0 to $\frac{x-u}{v-u}$. Next, at $x=v$, there's another step change from $\frac{x-u}{v-u}$ to $\frac{w-x}{w-v}$. Finally, at $x=w$, there's a final step change from $\frac{w-x}{w-v}$ back to 0. These step changes do not introduce infinite discontinuities; they are finite and predictable.

Therefore, the membership function of a TFN is continuous within each interval of its domain and has a finite number of discontinuities at the boundaries. This confirms that a TFN is piecewise continuous.

Step 5. Determine the alternatives' performance by eliminating each criterion.

Similar to the preceding step, this step also uses the logarithm function. The performance of the alternatives is determined based on removing each criterion separately in this step as opposed to *Step 4*. The computation is carried out using the following *Eq. (24)*.

Definition 13. Let m sets of performances and n criteria, $x_{ij}=(a_{ij},b_{ij},c_{ij})$, and Q'_{ij} indicate the alternatives' performance of ith alternative to the elimination of the ith criterion. The computation of this step is made using the following equation:

$$Q'_{ij}=(s_l,s_m,s_r) \tag{24}$$

Eq. (24), also known as Triangular Fuzzy Alternatives Performance, is a new definition obtained by integrating TFN with *Eq. (14)*. *Definition 13* is the same as in *Definition 12*.

Step 6. Determine the distance of each alternative.

The distance is calculated using *Eqs. (2)-(4)*.

Step 7. Determine the objective weights of the criteria.

The objective weights of criteria are calculated using *Eq. (16)*.

5 | Application of TFMEREC in MCDM

5.1 | Application of TFMEREC in Determining Criteria Weights

The proposed method, called TFMEREC, combines the principles of triangular fuzzy set theory and the MEREC method to facilitate decision-making in a fuzzy environment. TFMEREC aims to provide a comprehensive and reliable approach for assessing criteria and determining weights in MCDM scenarios. By utilizing linguistic variables represented as TFNs and employing normalization techniques, the decision matrix is transformed to ensure consistency and comparability. The overall effectiveness of alternatives is evaluated using fuzzy arithmetic, and the performance of alternatives is calculated by removing each criterion. The distances between alternatives are also computed using specific distance functions. Ultimately, the weights of criteria are determined, allowing for a systematic evaluation of the decision problem.

To demonstrate the application of TFMEREC in determining criteria weights, a simple decision matrix is utilized in this section. *Table 1* presents the opinions of three decision-makers (DM1, DM2 and DM3) for four different suppliers: S1, S2, S3, and S4. These alternatives have been evaluated based on four criteria, namely delivery timeliness (C1), Nonconformance rate (C2), product quality (C3), and pricing (C4). It is important to note that the attributes of C1, C2, and C4 are cost attributes, while C3 is a benefit attribute.

Table 1. Decision matrix.

	DM1 (W=0.35)				DM2 (W=0.25)				DM3 (W=0.40)			
Suppliers	**C1**	**C2**	**C3**	**C4**	**C1**	**C2**	**C3**	**C4**	**C1**	**C2**	**C3**	**C4**
S1	E	H	AL	VH	VH	AH	L	L	VH	AH	VL	VL
S2	L	L	AL	L	AL	VL	H	AH	SH	H	L	H
S3	AH	AH	H	VH	E	H	SH	L	AH	H	VH	VH
S4	L	VL	H	SL	VH	AH	VL	L	SL	VL	H	SL

Step 1. First create the decision-making matrix and, with *Table 2*, as a crucial reference. Furthermore, we divide the information from *Table 1* into three distinct tables, each to the specific needs of different decision makers.

Table 2. Linguistic variables and non-zero TFNs [67].

Linguistic Variable	Linguistic Scale
Absolute Low (AL)	(1,1,1)
Very Low (VL)	(1,2,3)
Low (L)	(2,3,4)
Slightly Low (SL)	(3,4,5)
Fair (F)	(4,5,6)
Medium High (MH)	(5,6,7)
High (H)	(6,7,8)
Very High (VH)	(7,8,9)
Absolutely High (AH)	(8,9,9)

Step 2. Create a decision matrix. Convert the linguistic variable provided by decision-makers in *Table 1* into a linguistic scale, then divide it into three distinct tables.

Table 3. Decision matrix DM1.

DM1				
	C1	**C2**	**C3**	**C4**
S1	(4,5,6)	(6,7,8)	(1,1,1)	(7,8,9)
S2	(2,3,4)	(2,3,4)	(1,1,1)	(2,3,4)
S3	(8,9,9)	(8,9,9)	(6,7,8)	(7,8,9)
S4	(2,3,4)	(1,2,3)	(6,7,8)	(3,4,5)

Table 4. Decision matrix DM2.

DM2				
	C1	**C2**	**C3**	**C4**
S1	(7,8,9)	(8,9,9)	(1,2,3)	(1,2,3)
S2	(5,6,7)	(6,7,8)	(2,3,4)	(6,7,8)
S3	(8,9,9)	(6,7,8)	(7,8,9)	(7,8,9)
S4	(3,4,5)	(1,2,3)	(6,7,8)	(3,4,5)

Table 5. Decision matrix DM3.

DM3				
	C1	**C2**	**C3**	**C4**
S1	(7,8,9)	(8,9,9)	(2,3,4)	(2,3,4)
S2	(1,1,1)	(1,2,3)	(6,7,8)	(7,8,9)
S3	(4,5,6)	(6,7,8)	(5,6,7)	(2,3,4)
S4	(7,8,9)	(8,9,9)	(1,2,3)	(2,3,4)

The next step is to multiply the values in these tables with their respective weights. As an illustration,

$$(S1, C1) = (4\times 0.35, 5\times 0.35, 6\times 0.35) = (1.4, 1.75, 2.1). \tag{24}$$

Table 6. Weighted decision matrix DM1.

DM1				
	C1	**C2**	**C3**	**C4**
S1	(1.4,1.75,2.1)	(2.1,2.45,2.8)	(0.35,0.35,0.35)	(2.45.2.8.3.15)
S2	(0.7,1.05,1.4)	(0.7,1.05,1.4)	(0.35,0.35,0.35)	(0.7,1.05,1.4)
S3	(2.8,3.15,3.15)	(2.8,3.15,3.15)	(2.1,2.45,2.8)	(2.45,2.8,3.15)
S4	(0.7,1.05,1.4)	(0.35,0.7,1.05)	(2.1,2.45,2.8)	(1.051.4,1.75)

Table 7. Weighted decision matrix DM2.

DM2				
	C1	**C2**	**C3**	**C4**
S1	(1.75,2,2.25)	(2,2.25,2.25)	(0.25,0.5,0.75)	(0.25,0.5,0.75)
S2	(1.25,1.5,1.75)	(1.5,1.75,2)	(0.5,0.75,1)	(1.5,1.75,2)
S3	(2,2.25,2.25)	(1.5,1.75,2)	(1.75,2,2.25)	(1.75,2,2.25)
S4	(0.75,1,1.25)	(0.25,0.5,0.75)	(1.5,1.75,2)	(0.75,1,1.25)

Table 8. Weighted decision matrix DM3.

DM3				
	C1	**C2**	**C3**	**C4**
S1	(2.8,3.2,3.6)	(3.2,3.6,3.6)	(0.8,1.2,1.6)	(0.8,1.2,1.6)
S2	(0.4,0.4,0.4)	(0.4,0.8,1.2)	(2.4,2.8,3.2)	(2.8,3.2,3.6)
S3	(1.6,2,2.4)	(2.4,2.8,3.2)	(2,2.4,2.8)	(0.8,1.2,1.6)
S4	(2.8,3.2,3.6)	(3.2,3.6,3.6)	(0.4,0.8,1.2)	(0.8,1.2,1.6)

Step 3. To combine all the decision matrix tables, we will need to aggregate all the decision matrixes from decision-makers. We use defuzzification.

$$\left(s_{11}^{l}, s_{11}^{m}, s_{11}^{u}\right) = \left(\min(s_{11}^{l}), \frac{\sum s_{11}^{m}}{3}, \max_{i}(s_{11}^{u})\right) = \left(\min(1.4, 1.75, 2.8), \frac{1.75+2+3.2}{3}, \max(2.1, 2.25, 3.6)\right)$$
$$= (1.4, 2.32, 3.6)$$

Table 9. Weighted defuzzification decision matrix.

	C1	**C2**	**C3**	**C4**
S1	(1.40,2.32,3.60)	(2.00,2.77,3.60)	(0.25,0.68,1.60)	(0.25,1.50,3.15)
S2	(0.40,0.98,1.75)	(0.40,1.20,2.00)	(0.35,1.30,3.20)	(0.70,2.00,3.60)
S3	(1.60,2.47,3.15)	(1.50,2.57,3.20)	(1.75,2.28,2.80)	(0.80,2.00,3.15)
S4	(0.70,1.75,3.60)	(0.25,1.60,3.60)	(0.40,1.67,2.80)	(0.75,1.20,1.75)

As an illustration, we will look at *Eqs. (17)* and *(18)* which is linear normalization in terms of both benefit and cost criteria:

Benefit criteria

$$n_{ij}(C3, S1) = \left(\frac{\min\ a_{ij}^{l}}{a_{ij}^{u}}, \frac{\min\ a_{ij}^{l}}{a_{ij}^{m}}, \frac{\min\ a_{ij}^{l}}{a_{ij}^{l}}\right) = \left(\frac{0.25}{1.60}, \frac{0.25}{0.68}, \frac{0.25}{0.25}\right) = (0.16, 0.37, 1.00).$$

Cost criteria

$$n_{ij}(C1,S1)=\left(\frac{a_{ij}{}^{l}}{\max\ c_{ij}{}^{l}},\frac{a_{ij}{}^{m}}{\max\ c_{ij}{}^{m}},\frac{a_{ij}{}^{l}}{\max\ c_{ij}{}^{l}}\right)=\left(\frac{1.40}{3.6},\frac{2.32}{3.6},\frac{3.60}{3.6}\right)=\left(0.39,0.64,1.00\right).$$

Table 10. The weighted normalized fuzzy decision matrix.

	C1	C2	C3	C4
S1	(0.39,0.64,1.00)	(0.56,0.77,1.00)	(0.16,0.37,1.00)	(0.07,0.42,0.88)
S2	(0.11,0.27,0.49)	(0.11,0.33,0.56)	(0.08,0.19,0.71)	(0.19,0.56,1.00)
S3	(0.44,0.69,0.88)	(0.42,0.71,0.89)	(0.09,0.11,0.14)	(0.22,0.56,0.88)
S4	(0.19,0.49,1.00)	(0.07,0.44,1.00)	(0.09,0.15,0.63)	(0.21,0.33,0.49)

Step 4. Calculate to determine the overall effectiveness of the alternatives.

In this step, a logarithmic scale with equal weights for the criteria is employed to assess the overall performances of the alternatives. Based on the normalized data acquired from the previous step, we can ensure that smaller values of s_{ij} lead to higher values of performances (Q_i). To demonstrate by example how to apply *Eq. (23)* when computing, we will perform some computational calculations using S1 and C1.

$$Q_1=\left(\ln\left(1+\left(\frac{1}{m}\sum_{k,k,\neq j}|\ln(c_{ik}^{x})|\right)\right),\ln\left(1+\left(\frac{1}{m}\sum_{k,k,\neq j}|\ln(b_{ik}^{x})|\right)\right),\ln\left(1+\left(\frac{1}{m}\sum_{k,k\neq j}|\ln(a_{ik}^{x})|\right)\right)\right)$$

$$=\begin{pmatrix}\left(1+\left(\frac{1}{4}\sum_{j}|\ln(1.00)|+|\ln(1.00)|+|\ln(1.00)|+|\ln(0.88)|\right)\right),\\ \left(1+\left(\frac{1}{4}\sum_{j}|\ln(0.64)|+|\ln(0.77)|+|\ln(0.37)|+|\ln(0.42)|\right)\right)\\ ,\left(1+\left(\frac{1}{4}\sum_{j}|\ln(0.39)|+|\ln(0.56)|+|\ln(0.16)|+|\ln(0.07)|\right)\right)\end{pmatrix}$$

$$=\left(0.03,0.50,0.92\right).$$

Table 11. The overall effectiveness of the alternatives (Q).

Q1	(0.03,0.50,0.92)
Q2	(0.34,0.77,1.15)
Q3	(0.46,0.63,0.88)
Q4	(0.26,0.76,1.12)

Step 5. Calculate the performance of the alternatives.

In this stage, each criterion is removed individually to determine how well the alternatives perform. If the performance of the ith alternative with respect to removing the ith criterion is represented by $Q_{ij}^{'}$. By eliminating each criterion, we can evaluate the performance of each alternative. When calculating C1, we will disregard it. The following are examples of computations using *Eq. (24)*.

$$Q_1^{'}=\left(\ln\left(1+\left(\frac{1}{m}\sum_{k,k,\neq j}|\ln(c_{ik}^{x})|\right)\right),\ln\left(1+\left(\frac{1}{m}\sum_{k,k,\neq j}|\ln(b_{ik}^{x})|\right)\right),\ln\left(1+\left(\frac{1}{m}\sum_{k,k\neq j}|\ln(a_{ik}^{x})|\right)\right)\right)$$

$$=\left(\begin{array}{l}\left(1+\left(\frac{1}{4}\sum_{j}|\ln(1.00)|+|\ln(1.00)|+|\ln(0.88)|\right)\right),\\ \left(1+\left(\frac{1}{4}\sum_{j}|\ln(0.77)|+|\ln(0.37)|+|\ln(0.42)|\right)\right),\\ \left(1+\left(\frac{1}{4}\sum_{j}|\ln(0.56)|+|\ln(0.16)|+|\ln(0.07)|\right)\right)\end{array}\right)$$

$$=(0.03,0.43,0.82).$$

Table 12. The performance of the alternatives (Q').

	C1	C2	C3	C4
S1	(0.03,0.43,0.82)	(0.03,0.46,0.86)	(0.03,0.33,0.72)	(0.00,0.36,0.61)
S2	(0.21,0.61,0.95)	(0.23,0.63,0.95)	(0.28,0.56,0.92)	(0.34,0.70,1.01)
S3	(0.44,0.58,0.79)	(0.44,0.58,0.78)	(0.09,0.28,0.59)	(0.44,0.55,0.71)
S4	(0.26,0.67,0.98)	(0.26,0.66,0.88)	(0.17,0.51,0.90)	(0.11,0.62,0.99)

Step 6. Calculate the distance using *Eqs. (14)-(16)*. The result will be shown in *Fig. 3*.

In this step, we will be using *Eq. (3)* to demonstrate the separation distance.

$$V_S=\sqrt{(a^m-a^l)^2+(b^m-b^l)^2+(c^m-c^l)^2}=\sqrt{(0.03-0.03)^2+(0.43-0.50)^2+(0.82-0.92)^2}=0.12.$$

Table 13. Separation distance.

Separation Distance				
E1	0.12	0.07	0.26	0.34
E2	0.29	0.26	0.32	0.16
E3	0.10	0.11	0.58	0.19
E4	0.17	0.26	0.35	0.25

In *Table 14*, all of the results have been summed to obtain the overall summation for the Separation Distance.

Table 14. Added separation distance results.

E1	0.12	0.07	0.26	0.34
E2	0.29	0.26	0.32	0.16
E3	0.10	0.11	0.58	0.19
E4	0.17	0.26	0.35	0.25
Sum	3.83			

Step 7. Calculate the weights of the criteria.

The weights of criteria are crucial when assigning importance or priority to various factors when making a decision or evaluating alternative courses of action. We will demonstrate some computation calculations using *Eq. (16)* as an illustration.

Next, we will demonstrate using W1 as an example.

$$W_j^{\circ}(W1) = \frac{V_J}{\sum_K V_k} = \frac{0.12 + 0.29 + 0.10 + 0.17}{3.83} = 0.18. \quad (1)$$

Table 15. The weight of the criteria.

	Weight
W1	0.178
W2	0.184
W3	0.394
W4	0.244

5.2 | Comparative Analysis

In this subsection, we conducted a comparative analysis to evaluate and compare the performance of different methods in determining criteria weights. Specifically, we compared the TFMEREC, TF CRITIC, and TF ENTROPY methods within the context of halal criteria. *Table 16* presents the weights obtained by each method for criteria W1, W2, W3, and W4.

The results highlight that each method yields distinct weights for the criteria. Notably, W3 holds the highest weight in both TFMEREC and TF CRITIC, while W1 and W2 exhibit comparable weights across different methods. The correlation coefficients reveal a positive association between TFMEREC and TF ENTROPY, indicating a close relationship in their weight assignments. Although the correlation with TF CRITIC is slightly weaker, it still demonstrates a positive connection.

TFMEREC distinguishes itself through its advanced algorithmic structure. Unlike traditional MCDM methods, which often rely on simplistic weight assignment techniques, TFMEREC employs a more sophisticated approach. By integrating cutting-edge algorithms, it enhances the precision and reliability of criteria weight calculations. Researchers and practitioners can benefit from this improved accuracy when making critical decisions.

In the decision-making scenarios, TFMEREC shines. While conventional methods may struggle to account for intricate interdependencies among criteria, TFMEREC excels. It provides a robust framework that effectively manages these interrelationships, ensuring consistent and reliable outcomes across diverse contexts. Decision-makers can confidently rely on TFMEREC to navigate complex scenarios.

This comparative analysis underscores the importance of considering different methods when determining criteria weights. The findings support the reliability and effectiveness of TFMEREC in assessing halal criteria weights, making it a valuable approach in MCDM scenarios.

Table 16. Comparable weights across different methods.

	TFCRITIC	TFENTROPY	TFMEREC
W1	0.2078	0.2443	0.1676
W2	0.2343	0.2458	0.1699
W3	0.2904	0.2623	0.4381
W4	0.2674	0.2476	0.2243

To further analyze the correlation between the results of TFMEREC and the other methods, we calculated the correlation coefficients shown in *Table 16*. These coefficients indicate the strength and direction of the relationship between the weights obtained from TFMEREC and the weights from TF CRITIC and TF ENTROPY methods. A higher correlation coefficient suggests a stronger correlation between the variables.

Table 17. The correlation coefficients of the comparative analysis.

	TFCRITIC	TFENTROPHY
Correlation coefficient, r	0.8510 $(p - \text{value} > 0.05)$	0.9960 $(p - \text{value} < 0.05)$

The intensity and direction of the association between the weights derived from TFCRITIC and TFENTROPHY techniques and the weights produced from TFMEREC are denoted by the correlation coefficients in *Table 17*. A greater correlation coefficient indicates that the variables are more strongly correlated.

The potential disparity in correlation coefficients between TFMEREC and TFENTROPHY could be attributed to the divergent approaches implemented by the two techniques. There is a possibility that TFMEREC is capturing more pertinent information or handling the complexity and ambiguity of the data more effectively, which could result in a more robust correlation with the weights.

Conversely, although TFENTROPHY remains efficacious, it may exhibit reduced proficiency in addressing these variables, hence yielding a diminished correlation coefficient. There is also the potential for greater correlation coefficients if TFMEREC's way of allocating weights is more closely aligned with the Fuzzy CRITIC and Fuzzy Entropy methods.

5.3 | Sensitivity Analysis

Sensitivity analysis plays a critical role in assessing the reliability and robustness of decision-making models. In the study on the TFMEREC approach for deriving criteria weights, we ran a sensitivity analysis utilizing three different normalization methods and three different distance methods. By analyzing different normalization and distance methods, we aimed to evaluate the TFMEREC method's response to variations. This sensitivity analysis aimed to study how different methodological choices influence criteria weights and the decision-making process.

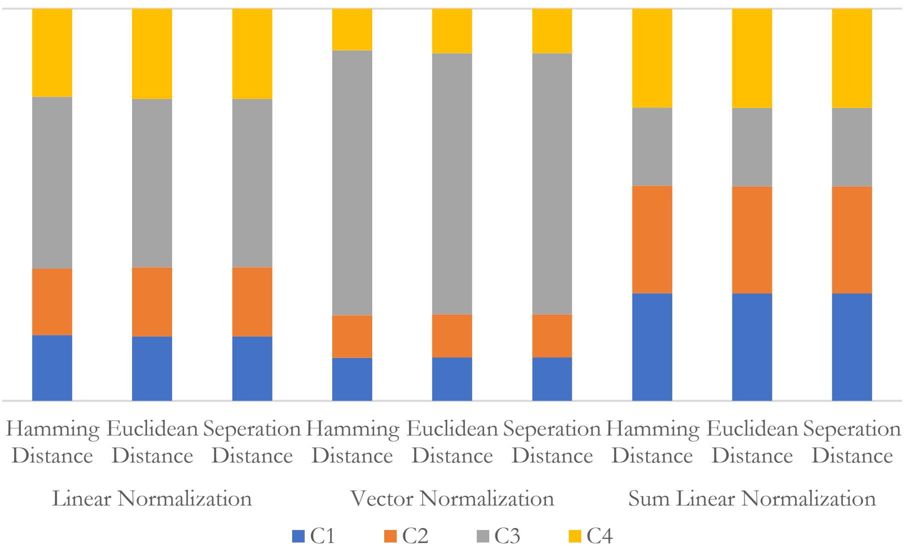

Fig. 3. The weight of criteria with different normalization and distance techniques.

The analysis depicted in *Fig. 3* demonstrates that the fluctuation pattern of criterion weights in TFMEREC is influenced by the choice of normalization method and distance method, both of which were evaluated in the sensitivity analysis. Notably, the weights calculated using TFMEREC exhibit a robust correlation with the weights obtained through Linear normalization, Vector normalization, and Sum Linear normalization. Considering MCDM scenarios where linear normalization is employed, the results generated by TFMEREC yield weights can be deemed acceptable and trustworthy.

6 | Conclusion

In summary, while TFMEREC provides a robust framework, ongoing research efforts can refine its capabilities, scalability, and adaptability to meet the evolving needs of decision-makers. The proposed TFMEREC method improves decision-making processes by addressing the inherent ambiguity and uncertainty in real-world circumstances. To begin, it outlines a detailed procedure for determining criteria weights, recognizing the multifaceted nature of decision-making. Unlike simplistic approaches that ignore nuances, TFMEREC provides a strong framework for decision-makers by combining TFNs and utilizing the MEREC approach. This systematic methodology walks people through the process of determining the importance of criteria, taking into account interdependencies and arriving at meaningful weightings.

Furthermore, TFMEREC uses linguistic variables and TFNs. Linguistic terms are extremely important in practical circumstances. People use terms like very important, somewhat relevant, and slightly favorable" to describe their preferences, opinions, and uncertainties. TFMEREC embraces this language richness, enabling effective judgmental communication. Furthermore, the use of TFNs broadens the usual crisp numerical method, capturing the nuances of imprecise information.

Next, to ensure comparability and consistency across diverse criteria, TFMEREC employs normalization techniques. Decision matrices often contain criteria measured on different scales. Normalization creates a level playing field, enhancing the reliability of resulting weights. Whether it's linear normalization, vector normalization, or sum linear normalization, the goal remains consistent: fair assessments of different alternatives and their overall efficacy.

TFMEREC goes beyond traditional methods by accounting for individual criterion removal. In dynamic decision-making environments, criteria can change or become irrelevant. By systematically evaluating the impact of removing specific criteria, decision-makers gain insights into the robustness of their choices. This adaptability is crucial when dealing with evolving scenarios or unforeseen changes.

The method's validation lies in its ability to withstand scrutiny. TFMEREC undergoes comparison with other methods, such as TF CRITIC and TF ENTROPY. The significant correlation observed reinforces its dependability. Decision-makers can trust that the weights assigned by TFMEREC align with established methods, enhancing their confidence in subsequent decisions.

Furthermore, sensitivity analysis investigates how different normalization and distance procedures affect criterion weights. TFMEREC displays robustness by retaining significant correlations across several normalization methods. This versatility guarantees that decision-makers can rely on it even when circumstances change.

The TFMEREC framework provides a robust platform for decision-making, but it also contains limitations and areas for further investigation. While TFMEREC is robust, there are certain restrictions to consider. As decision settings get more complicated, the scalability of TFMEREC may be tested. It is critical to investigate methods for dealing with large-scale decision problems rapidly while maintaining accuracy. TFMEREC uses data to calculate criteria weights and do comparisons. In practice, collecting precise data might be difficult. Addressing data scarcity and imprecision is a continuing challenge. The language factors in TFMEREC introduce subjectivity. Different decision-makers may interpret words like "very important" differently. Balancing subjectivity with objectivity is critical. While TFMEREC takes into account interdependencies, there is still room for improvement in capturing complicated interactions among criteria.

In terms of future research directions, there are various options to examine. First, create adaptive algorithms to alter TFMEREC's settings based on the problem environment. This would improve its adaptability to shifting conditions. Hybrid approaches: Investigate hybrid approaches that integrate fuzzy logic with other decision-making techniques (such as machine learning and neural networks). Such hybrids may increase accuracy and robustness. Improve TFMEREC's handling of uncertainty. Investigate sophisticated fuzzy modeling strategies for better representing imprecise data. Involve decision-makers in the design process.

Their observations can help steer modifications and ensure practical usage. Conduct in-depth case studies across multiple areas to confirm TFMEREC's performance and identify unique issues. In summary, TFMEREC is a valuable tool, but ongoing research can refine its capabilities and address its limitations, making it even more effective for decision-makers in complex environments.

Finally, TFMEREC extends over theoretical frameworks and serves as a practical tool for decision-makers exploring a complicated environment. Its versatility, robustness, and sophisticated handling of ambiguity make it a significant asset in the decision-making toolset.

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Author Contribution

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Conflicts of Interest

The authors declare no conflict of interest.

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